**Exam. Code : 211002**
**Subject Code : 5541**

**M.Sc. (Mathematics) 2nd Semester**

# TENSORS AND DIFFERENTIAL GEOMETRY

**Paper—MATH-562**

Time Allowed—Three Hours] [Maximum Marks—100

**Note :—** Attempt **TWO** questions from each unit. All questions carry equal marks.

## UNIT—I

1. Define Certesian Tensor of order 4. Also define contraction and state and prove contraction theorem.

2. Show that $\delta_{ij}$ is a tensor of order two.

3. Show that the transformation of a mixed tensor possess the transitive property.

4. Show that Christoffel symbols do not behave like tensor.

## UNIT—II

5. Define principal normal and binormal. Find the equations of the principal normal and binormal.

6. State and prove Serret-Frenet formulae.

7. Find the curvature and torsion of the curve

$$x = a(u - \sin u),\ y = a(1 - \cos u),\ z = bu.$$

8. Find the centre and radius of spherical curvature.

## UNIT—III

9. Investigate the spherical indicatrices of the circular helix $x = a \cos \theta$, $y = a \sin \theta$, $z = c\theta$, $c \neq 0$.

10. Find the envelop of the plane $lx + my + nz = 0$ where $al^2 + bm^2 + cn^2 = 0$.

11. Find the condition that the surface given by $z = f(x, y)$ may be developable.

12. Calculate the fundamental magnitudes to the surface $2z = ax^2 + 2hxy + by^2$ taking x, y as parameter.

## UNIT—IV

13. Define conjugate direction. Find an analytic expression for two directions to be conjugate.

14. Show that the necessary and sufficient condition that the parametric curves be lines of curvature are $F = 0$, $M = 0$.

15. Find the asymptotic lines on the surface $z = x \sin y$.

16. State and prove theorem of Beltrami and Enneper.

## UNIT—V

17. Show that the curves u + v = constant are geodesics on the surface with metric

$$(1 + u^2)\, du^2 - 2uv\, du\, dv + (1 + v^2) dv^2.$$

18. Show that geodesic curvature vector of any curve is orthogonal to the curve.

19. State and prove Gauss – Bonnet theorem.

20. Find the condition that surface s may be mapped conformally onto surface s'.